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Mathematics News Letter

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This journal is dedicated to mathematics in general, to the following causes in particular (1) the common problems of grade, high school and college mathematics teaching, (2) the disciplines of mathematics, (3) the promotion of Mathematical Association of America and National Council of Teachers of Mathematics projects.

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THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

The 88th annual meeting of the greatest scientific association in the world will be held in New Orleans during the last days of December 1931 and the first days of January, 1932, the session probably lasting one whole week. Many years have elapsed since an annual meeting of the A. A. A. S. came to Louisiana. Last year it was held in Cleveland, Ohio, the estimated attendance of scientists being not less than 10,000. Four years ago Nashville, Tennessee, entertained the great scientific body, with an attendance which, we believe, was estimated at five or six thousand.

The deep significance which this meeting in New Orleans of the world's scientists, now less than six months away, has for scientific advance in the south, especially in Louisiana, is only partially shown in the following list of the grand divisions of A. A. A. S., each of which has its own organized separate program: Mathematics, Physics, Chemistry, Astronomy, Geology and Geography, Zoological Sciences, Botanical Sciences, Zoological and Botanical Sciences, Anthropology, Psychology, Social and Economic Sciences, Historical and Philological Sciences, Engineering, Medical Sciences, Agriculture, Education, Science in General.

The order of these divisions as here printed is just as it is given in the published "GENERAL PROGRAM" of last year's Cleveland meeting, Mathematics being called division "(A)", Physics, "(B)", Chemistry, "C)", etc.

Readers of the News Letter are properly to be assumed to have a leading interest in division "(A)", or the mathematical division. Most of them already know that it is composed of the Mathematical Association of America and the American Mathematical Society, the former representing the interests of collegiate mathematics and a certain amount of research activity, while the latter is a strictly research organization. The two organizations taken together contain a total membership of several thousand mathematical teachers and research workers.

The concentration of America's strongest mathematical men and women in the New Orleans programs of these two great organizations, taken with the actual presence of these leaders in our midst, is a stirring professional challenge to Louisiana's mathematical workers.

Shall we accept the challenge? AND HOW?

—S. T. S.

THE MATHEMATICS NEWS LETTER—A CHALLENGE!

With this issue, the April-May double number, Nos. 7-8, is closed Vol. 5 of the Mathematics New Letter. We have no apologies to offer for the delay which has prevented its being printed and distributed at the time it was due to be printed and distributed. However we do profoundly regret the causes which have forced such a delay. Those who have read our financial statement published in the last issue will appreciate the nature of these causes. It is our profound hope that some way shall yet be found to crystallize an effective support—financial and otherwise—of a journal which is devoted purely and simply to the cause of mathematics and its teaching, particularly in the states of Mississippi and Louisiana. We believe that such a cause is entitled to some measure of assistance from every high school, grade and college teacher of mathematics in the Louisiana-Mississippi territory! We have more faith than ever in the PROFESSIONAL IDEA which the New Letter stands for, namely,

that scientific cooperation and correlation between secondary and college mathematics is profoundly needed if mathematics is ever to assume its deserved place of pre-eminence in the schools of the world.

The fact that, latterly, has developed, apparently, a wider divergence between the aims of secondary education and some of the heretofore generally accepted aims of collegiate training only adds to the obligation of all teachers of mathematics, from the grade to the college, to preserve a well-knit solidarity of purpose, in order that the dangers arising from this wider divergence may be avoided.

The News Letter is a CHALLENGE to every mathematics teacher in our two States.

—S. T. S.

THE HEURISTIC METHOD VERSUS THE LECTURE METHOD IN DEVELOPING REFLECTIVE THINKING

By DORA M. FORNO,
New Orleans Normal School

Developments in recent educational history show an amazing change in the conception of the purposes of high school education. Formerly, the high schools were organized for the youths who expected to go to college; today, there is a recognition of the needs of pupils not going to college, and "high school training for all children" is emphasized.

Social efficiency has been adopted as the ultimate aim in high school education, but social needs vary in different communities and we, therefore, have various interpretations of what is to be the curriculum that shall develop this social efficiency. It is generally conceded that information is the proxmate aim that is most prominent in high school instruction. Information is important, but it is subsidiary to the other psychological aims in education; namely, habits and skills, attitudes and ideals, and many-sided interests.

The teacher's methods of instruction are determined largely by the aims which she has set up as the goals of education.

If information is the goal, the lecture method will be the

chief mode of instruction and the study lessons will be characterized by the memorization of rules, formulas and theorems. The lecture method is to be condemned rather than encouraged in the high school, especially in the teaching of mathematics. Only such subject matter that is not readily developed inductively should be imparted by lecture.

If by social efficiency is meant efficiency which calls for reflective thinking, for the ability to make abstraction and generalizations, and for self-reliance and confidence in ones ability to accomplish a self-task, then the Lecture Method must give place to the Heuristic Method.

The Heuristic Method is derived from a Greek work euriskein which means "to discover". The purpose of this method is to stimulate the pupil to reflective thought that he may rediscover the underlying principle, formula or proof for himself. The mode is inductive. The problem involves a situation familiar to the groups, and the class is led along by skillful questioning until the desired end is attained. If right attitudes and ideals have been developed, the pupil is eager to think out the situation for himself and gain the necessary end. The aim in this method is not the giving of facts but the development of power through self-expression. The active cooperation of the pupil is enlisted and the teacher elicits from the pupil the desired method of attack and procedure by recalling similar situations or known facts that will help in attaining the desired results, always steering in the right direction and not allowing any divergence from the big problem confronting the class. Intelligent inferences will be made by the class which are regarded as tentative until they are verified by subsequent facts or by reference to the text-book.

The Heuristic Method provides the four types of opportunities to do reflective thinking which Parker has classified as follows:

1. "Opportunities for the student to do his own reasoning, that is, to reason independently."

2. "Opportunities to follow and supplement the reasoning of other students."

3. "Opportunities to follow and supplement the reasoning being done by the teacher".

4. "Opportunities to follow and supplement the reasoning set forth in a book".

"Honest-Thinking the Goal" should be the purpose of all mathematics teaching and the heuristic Method is charaterized by this aim, in fact honest-thinking is the backbone of the Heuristic Method.

The Heuristic Method of development makes possible truly independent study, for the pupil has acquired a method of study, and "Supervised Study" has become something more than just a name.

In conclusion, I shall quote in full from Parker's methods of teaching in the High Schools, pages 199-200, for suggestions for guiding reflective thinking. Parker says, "These rules may be briefly summarized as follows:

"To stimulate and assist pupils in carrying on reflective thinking the teacher should

I. Get them to define the problem at issue and keep it clearly in mind.

II. Get them to recall as many related ideas as possible by encouraging them.

(1). To analyze the situation and

(2). To formulate definite hypotheses and to recall general rules or principles that may apply.

III. Get them to evaluate carefully each suggestion by encouraging them:

(1). To maintain an attitude of unbiased, suspended judgment or conclusion.

(2). To criticize each suggestion.

(3). To be systematic in selecting and rejecting suggestion, and

(4). To verify conclusions.

IV. Get them to organize their material so as to aid in the process of thinking and encourage them:

(1). To take stock from time to time.

(2). To use methods of tabulation and graphic expression, and

(3). To express concisely the tentative conclusions reached from time to time during the inquiry.

A HIGH SCHOOL MATHEMATICS CLUB

By HATTIE C. GARRETT,
Baton Rouge, La.

It has been observed by the writer as a teacher of high school algebra and geometry, that most pupils take these subjects just because they are prescribed, and not because of any vital interest in, or recognized need for, acquiring further knowledge of mathematical relations and applications. Feeling that this condition should be corrected, if possible, we investigated a great many devices recommended by educational leaders. Finally the idea of a voluntary organization, in the form of a Mathematics Club, was seized upon as having more to offer than any other single plan. It is the purpose of this paper to outline, the planning, organizing, and functioning of the club through the first year of its existence.

Lacking previous experience in handling such extra-curriculum activities, it at once became obligatory that the sponsor explore the literature in the fiield of high school club-work, select outstanding contributions, and do a great deal of reading, note-taking, and planning.

Among the best books examined were the following:

Blackburn, **Our High School Clubs,** Macmillan Co., New York, 1928.

Hassler and Smith, **The Teaching of Secondary Mathematics,** Macmillan Co., 1930.

Thomas-Tindal and Myers, **Junior High School Life,** Macmillan Co., 1924.

The most useful periodical found was the Mathematics Teacher.

Out of this bit of research certain definite conclusions were reached. Briefly outlined, they are as follows:

1. Clubs can be mere time-fillers, or they may be so planned and systematized as to enrich the interests of superior pupils, and strengthen the weaknesses of subnormal pupils.

2. Clubs offer opportunities for social guidance and training for citizenship not found in regular class recitation.

3. Clubs should be mainly serious in their objectives, but should afford some form of wholesome recreation.

4. Club organizations lend themselves to vocational guidance.

5. Membership should be entirely voluntary and have no direct bearing upon class standing.

6. A mathematics club offers opportunity for studying the history of the development of mathematics, and allows pupils to become acquainted with the fundamental concepts of the number system.

7. Every club must justify its existence by a specific aim.

8. The sponsor must expect to do a lot of hard work to meet some disappointments. She should be willing to remain a guiding hand, placing the burden of the responsibility upon the members.

9. The club should so function as to be a real influence in the school.

10. The details of organization, such as drafting of the constitution, by-laws, etc., should be kept definite and simple, but every club should have them.

Informally, in the course of regular classwork one day, the sponsor presented the club idea. No mention was made of organization, as it was hoped that this suggestion might come from the pupils themselves. When the pupils learned that whole volumes had been written on the subject of high school clubs, and that often several clubs existed in a single school, they became impressed with the value of the idea, and began to ask many questions. The sponsor told them of such a club to which she had belonged, and made accessible to her classes several very interesting articles on high school mathematics clubs. Those articles concerned with mysterious problems, mathematical puzzles and recreations and with the history of the subject seemed to make the strongest appeals.

Several days later, a boy in the junior class asked why we couldn't have a club. So all junior and senior mathematics students were asked to bring in next day, in outline form, their reasons for wanting, and for **not** wanting a mathematics club. Stress was placed upon the idea that both negative and affirmative evi-

dence was sought, for no club should be organized if it could not prove its worth to the pupils themselves.

Most of the class time for one day in all junior and senior sections was devoted to floor talks by the pupils. There was some debate, of course, but the outcome showed a very general interest in the proposed club, and a strong feeling on the part of the pupils that it would be highly worthwhile.

Having given the pupils ample opportunity to react, the sponsor then outlined her ideas on the subject. Summarized, they are given below:

1. Membership must be voluntary and have no bearing on grades.
2. The act of becoming a member should signify the pupil's intention to boost and support all activities of the club.
3. Details of organization must be definitely settled, and approved by sponsor and principal before being set in operation.
4. At the conclusion of each program light refreshments might be served, but the cost must be held down to 10c per member, and never allowed to eclipse the real purpose of the club.

When several days had elapsed without further mention of the club, the sponsor called a meeting requesting the attendance of all pupils interested in the organization of a mathematics club. More than half the junior and senior enrollment responded. The principal, who aided very greatly in untangling several details as to time and place for meeting, was also present. Officers were chosen, name and motto selected, and a committee appointed to draw up the constitution. Another committee was appointed to plan the first program, to be approved by the sponsor. It was also decided to have a new program committee each time, thus allowing more members to take part.

It was decided to fix the dues at twenty-five cents a year, using the proceeds in building up a much needed departmental library. If other expenses should arise, they were to be met by pro-rating the amount among the members, or by the proceeds from plays sponsored by the club apart from regular meetings.

The principal suggested that one meeting be held near the close of each six weeks period, and offered the club the use of the

study hall. His offer was gratefully accepted, and meetings were held there except when field trips had been planned.

The first program is given because it may prove suggestive, and is quite typical.

1. How We Got Our Number System.
2. March of the Mathematicians.
3. Puns from Geometry.
4. Address by Professor of Mathematics, Louisiana State University.

The school is extremely fortunate in being very near the State University, and in having the active co-operation of the mathematics staff, three of whom addressed the club on different occasions during the year. At another time a member of the State Department of Education led a very interesting discussion on **Mathematics as the Language of Science.**

As a grand finale, the club planned an excursion lasting through one whole afternoon. Leaving school at the noon dismissal, the members went to the city park where a picnic dinner was enjoyed, and the zoo visited. At one o'clock the members went to the University campus and listened to a very practical lecture by the Dean of the College of Engineering on the relation of high school mathematics to engineering. After the lecture, a tour was made of the campus which took in the shops, stadium, and Greek Theatre. The peculiar problems in mathematics which confronted the engineer in building these structures were pointed out, and the method of their solution outlined.

At the close of the school year there was evidence that the "Math Club" will be very much alive when school re-opens in the fall. It threatens to tax the sponsor's wits to keep it from becoming too large and unwieldy, which is as great a fault as being too small.

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MATHEMATICS CLUB ACTIVITIES
A. AND M. COLLEGE, MISSISSIPPI

Reported by C. R. STARK

Early in the session of 1930-31 the Mathematics Staff in discussion with Dr. C. D. Smith, head of the department, had under

consideration the problem of presenting mathematics to students from a more vital and interesting point of view. Largely at the suggestion of Dr. Smith it was decided to organize among the more successful students at A. and M. a Mathematics Club which in some respects differs from clubs now known by this name. The history of such clubs is too often a brief statement of membership for all, too frequent meetings with impromptu programs, lag of interest, and finally a state of neglect. It is believed that a club which features the major interest of members and the practical application of mathematics as an extra curricular activity will not suffer this fate.

The preliminary steps in organization and the list of initial objectives may be briefly indicated as follows: A list of charter members was selected by the staff from those who ranked in the upper five per cent of those enrolled in advanced classes. This group was assembled and the plan placed before them. The vote to organize followed and a temporary organization was set up. A list of candidates was prepared by the charter members from those who ranked high in advanced mathematics and the meeting was planned to effect the permanent organization. With such personnel the club is assured of capable and enthusiastic members. It is assumed that those of mediocre record with other subjects as major interest will not profit by a mathematics club but rather will serve to defeat the purpose of club work thru lack of interest. Initial objectives were set up as follows:

1. To support a more useful mathematics program at the college.
2. To seek talented students who should develop special abilities in mathematics.
3. To study the relations of mathematics to science and industry.
4. The solution of special problems.
5. Preparation of papers on mathematical topics during spare time.
6. To assist in preparattion of indices for library reference in mathematics.
7. Service to the public in matters of elementary mathematics.
8. To cultivate social contacts and the free exchange of ideas.

With these objectives in mind the fall meeting was devoted

to organization at which time the constitution and by-laws were adopted and coommittees appointed. The officers in charge were continued for the year. At the regular meeting of the Winter Quarter we had two interesting papers from student members, the subjects being The Use of a Reference Library, and On the Origin of Arithmetic and Algebra. During the Spring Quarter members of the club made a visit to the Mathematics Club at the Mississippi State College for Women, Columbus, Mississippi. By invitation a program was given on plans for club organization. It was decided at this meeting that the two clubs should have a joint annual session. At the Spring meeting two papers were given by students, the one on The Use of Graphs in Chemistry, and the other on A Problem of Maximum Displacement. At each meeting a round table discussion is conducted. Altho any topic of interest may be heard at this time by the leader the discussion is usually focused in the main on a topic that has been previously announced.

The interest and special work of members to date is most gratifying. Especially is this shown in the interest of the freshmen who find that they have been selected by their fellow students for their superior accomplishments in mathematics. Such indicates that initial objectives of the club will be realized in the near future. The feature of this type of organization lies in the fact that the program of activities is left open to challenge the membership and develop leadership in many ways.

In order to give a definite picture of the organization we include here a copy of the constitution and by-laws.

MATHEMATICS CLUB OF THE A. AND M. COLLEGE

CONSTITUTION

Article 1—Name.

The name of this club shall be the Mathematics Club of the A. and M. College.

Article 2—Purpose.

The main purpose of this club is to stimulate interest in mathematics, to offer opportunity for special studies and reports regarding the uses and applications of mathematics, to promote closer and more helpful relation between members of the staff and students interested in mathematics, and to provide for assistance to those interested in the study of special topics.

Article 3—Membership.

Section 1. Membership shall be classed as: active, associate, honorary, and alumni who were active members at the time of graduation.

Section 2. Charter members in good standing and students who may be elected upon recommendation of the membership committee, are called active members.

Section 3. Associate members shall consist of members of the freshman class who may be elected upon recommendation of the membership committee. Such members shall have all rights and privileges of the club except the right to vote and hold office.

Section 4. Honorary members shall consist of members of the mathematics staff and others who may be elected by unanimous vote of the club.

Section 5. Alumni members shall consist of those who are active members at the time of graduation.

Article 4—Officers and Their Duties.

The officers of this club shall be President, Vice-President, and Secretary, elected annually at the regular meeting of the fall quarter. The duties of President and Vice-President are those usually delegated to such officers. The Secretary in addition to the usual duties acts as reporter for the club.

Article 5—Meetings.

Regular meetings shall be held on Wednesday prior to the week which precedes the regular examinations. Twelve active members shall constitute a quorum.

Article 6—Standing Committees.

The standing committees are those on membership, arrangements, and program. Each committee shall consist of three members appointed by the President to serve during his administration. Their duties shall be as indicated by the name of the committee.

Article 7—Departmental Advisor.

A special advisor shall be appointed each year by the head of the mathematics department. In this way a member of the staff is available for special assistance to members of the club.

Article 8—Amendments.

This constitution my be amended at a regular meeting by a two-thirds vote of the members present, provided that such shall constitute a quorum. Proposed amendments shall be presented in writing by an active member of the club at least one regular meeting prior to the ballot.

BY-LAWS

Article 1—Order of Business.

1. Call to order by the President.
2. Reading of minutes of previous meetings not yet approved.
3. Report of standing committees.
4. Report of other committees.
5. Reception of new members.
6. Unfinished business.
7. New business.
8. Election of officers.
9. Appointment of committees.
10. Program.
11. Adjournment.

Article 2—Voting.

Officers and members shall be elected by secret ballot. Vote on routine matters shall be indicated at the discretion of the President. Those proposed for membership are not elected if more than one vote is cast against them. In all other cases the question is carried by majority vote. Each active member shall have one vote.

Article 3—Qualifications for Membership.

Students are eligible who elect courses in mathematics in addition to those prescribed for freshman, and who have maintained an average above 85 in mathematics. Members of the freshman class to be eligible for associate membership must maintain an average above 85 in mathematics for the first two quarters.

Article 4—Amendments to By-Laws.

Amendments shall be made in the manner prescribed for amendments to the constitution.

A BIT OF HISTORY*

By W. PAUL WEBBER,
Louisiana State University

There is little in mathematics that is of popular interest. There is little of the spectacular. There is little that can be made a topic of conversation at a social gathering. This does not mean, however, that mathematics is without human interest. Mathematics is in itself as simple as any one of several other sciences, but it is so constituted that one who would appreciate its beauties must undergo long training, beginning with the elements and gradually reaching the higher parts, wherein are symmetry, imagination, discrimination and beauty. Many of the branches of mathematics have been called into existence by the needs of scientific research and in some cases a branch of mathematics has been devoted as pure research and has afterward been found essential to the solution of some scientific problem. As an example of the latter we cite the theory of quadratic differential forms which was developed some years before Einstein proposed his theory. This very theory of quadratic differential forms was precisely what Einstein needed to develop his new mechanics.

The art of calculation in some form or other has always been necessary, wherever men have lived together in communities.

These methods were simple at first, but gradually became more scientific as civilization became more complex. History tells us that ancient peoples had attained quite an advanced stage in the art of numbering, but that their knowledge could not be introduced into western nations until after the "dark ages," that is, until the revival of learning about a thousand years after the fall of the Western Empire of Rome.

When men in Europe began to study mathematics seriously there was rapid progress, and there came many treatises on arithmetic and its applications. Most of these were of quite limited scope, (but quite sufficient unto their day and time). They dealt chiefly with commercial problems. The socially elite con-

*The writer is indebted to the writings of Professor Cajori and others for the material herein set forth. Thirty years teaching, several of which were in the old one room school may have affected some statements.

sidered arithmetic (mathematics) as vulgar and only for the mercenary and for mechanics. With the methods of travel and communication then available progress in learning was disseminated slowly. A few terms have come down to us as a result of the methods of calculating, employed in these primitive days. For example, the word "exchequer" still used in connection with English treasury matters had its origin in the fact that government accounting was conducted on a table covered with a chequered cloth, by moving blocks from one position to another on this cloth. Here is the origin of Chancellor of the "Exchequer."

In the boyhood years of the writer a similar device was used for keeping tally of the number of bushels of grain threshed at the threshing machine. It consisted of a tally board in which holes were bored. Tally was kept by placing pegs in these holes and moving a peg for every bushel as it came from the thresher. The writer knows of one instance where men were splitting fence rails for a farmer and they kept tally by breaking off a piece of twig for each rail split. At the end of the day they counted twigs.

The art of numbering as we know it, is the result of three inventions, namely: The Hindu notation, with its position value, and zero, decimal fractions, and logarithms. These appear simple and commonplace to us, but in their day these inventions were momentous. Without them, or some equivalent of them, science and invention must have developed much more slowly than they did and we might have been born in the seventeenth century, scientifically speaking, instead of the twentieth.

The first of the three inventions above cited was in ancient times, but was not introduced into Europe until about 1200, A. D. The other two inventions are modern. The invention (or reinvention) of decimal fracations is credited to Simon Stevin whose work was published about 1584. John Napier's invention of logarithms came nearly a century later for Napier was born in 1630. When Professor Henry Briggs of London learned of Napier's invention he made the trip to Scotland to see Napier. When they met, so great was their emotion, that they could not speak for several minutes. It is of interest to note that Briggs' trip was as great an undertaking at that time as a trip from

New York to San Francisco is today, if not greater. Briggs suggested an improvement to Napier, but the inventor had a better one to which both agreed, and it was further agreed that Briggs should arrange the logarithms and have them published. These are the logarithms now commonly used in calculating.

The invention of logarithms came in the niche of time. For Kepler was working on his planetary theory, and the Germans had completed elaborate tables of the trigonometric functions. Logarithms were needed to expedite the labor of computing. Laplace said the invention of logarithms doubled the life of the astronomer by reducing his labors. At that time an enormous amount of calculating was necessary, as astronomy was becoming an exact science and orbits and motions of the heavenly bodies were to be calculated.

The slide rule which is a "table of logarithms on a stick" was originally called Napier's rods, decimal fractions and logarithms. We should note that the invention of logarithms followed close on the introduction of the Hindu notation into England, which did not occur until about 1500.

The machinery of calculation being now available, progress in other branches of mathematics would naturally be rapid.

We shall now turn to another phase of our subject and find scenes of a very different character. We shall take a look at the mathematical situation in our own country in colonial times. This will be immediately following the situation in Europe just described. For lack of a better name we shall call it a chapter from the teaching of mathematics in the United States. In colonial days teaching was, more than now, not a profession in the true sense of the word, but rather a stepping stone to some real profession or career. The colonial schoolmaster was hired to teach the children to read, write and sometimes to cipher. He was an exceptional teacher if he possessed a fair knowledge of fractions and the rule of three, i. e., proportion. If some brilliant pupil exhibited such rare genius as to master fractions or to pass by the rule-of-three he was accounted a finished mathematician. It is not stated whether he was called professor or doctor. The best teachers of colonial days were college students or graduates who taught while their minds attained sufficient maturity to enable them to enter business or some profession.

The little salary paid them no doubt was an advantage to them in paying college expenses. While the situation is much improved, we have still something to do in this line, even today.

John Adams who afterward became president of the United States was a graduate of Harvard. After graduating and before studying law he had charge of the Grammar School at Worcester, Mass. The following extract from a letter of his, dated Sept. 2, 1775, may be of interest. "As a haughty monarch ascends his throne, the pedagogue mounts his awful great chair and dispenses right justice through his whole empire. His obsequious subjects execute imperial mandates with cheerfulness and think it their high happiness to be employed in the services of the emperor. Some times paper, some times knife, now birch, now arithmetic, now ferule, then scolding, then flattery, then thwacking, then A, B, C, calls for the pedagogue's attention." How can a teacher read this without feeling kinship with Adams? He must have had a sense of that saving grace, humor, or he could not have written in such phrase. The tale of Ichabod Crane is hardly an overdrawn sketch of that time. Indeed I am told that it would still be possible to find at this date in remote and backward communities a fair duplicate of the above picture in all essential particulars.

About the only apparatus available to the colonial school teacher were the birch bark for writing, and the birch rod for another kind of "righting," and the historic ferule. These instruments are still used in communities where the "sweetly draw them out" policy or Montessori, or some more recent humane method, is not sufficiently persuasive to motivate the stubborn wills of the pupils.

The backs of old spelling books, the fly leaves of ledgers, and every thing that could supplement the birch bark were pressed into service for doing sums. Text books of arithmetic were rare in America at first, and by no means did every teacher possess one in printed form. Most teachers had ciphering books in which was recorded all they knew or ever hoped to know of mathematics, carefully written out in their own hand. This mathematical knowledge, like Gaul of Old, consisted of three parts: the sums, the rules, and the solutions of the sums by the rules. When teaching, the master dictated the sums and the

rules which the pupils copied. It was then the pupils' business to do the sums by the rules. There was no regular class work in arithmetic. If the pupil failed to solve any sum he went to the master who solved the problem for him, without comment or explanation, the pupil copied this and his own solutions to all problems in his ciphering book. This was considered as completing the teaching process. Often a line of a dozen would stand waiting their turn to get help from the master. The master rarely explained, but only gave the rules and solved the problems that were too difficult for the pupils. Some of the ciphering books bear evidence of much care in preparation.

It must be admitted that whether on account of the method or in spite of it, a number of skilled arithmeticians were developed. We know that some students will make good no matter what kind of a school they attend, some will never do so under the most favorable conditions, while a larger number will be quite seriously affected by the school which they attend. This fact alone would justify some attention to methods of teaching and of school administration.

Arithmetic was rarely taught to girls in colonial times. They probably did not need it. Many have little or no use for it today. However, some women have become very respectable mathematicians, showing that the time honored distinction of mathematical ability by sex is not a universal characteristic of the race, even though some girls declare that mathematics and particularly algebra should be omitted from the course of study. Indeed some men of recent times have so endorsed this policy as to assert that "algebra is the last and most diabolical of the dynasty of soul killer." A little analysis of the evidence on which these conclusions are based would show that even these wise men might have been more modest in their assertions if they had possessed some real knowledge of the object of their invective. In one college, where the writer attended, a woman was professor of mathematics, including analytic mechanics and descriptive geometry. Her college won a gold medal at the World's Fair in 1893, on "Students work and geometrical contrivances."

Bronson Alcot, educator, born in Massachusetts in 1799, thus describes the schools of his boyhood: "Until within a few years,

no studies were permitted except spelling, reading and writing. Arithmetic was taught by a few instructors one or two evenings a week. But in spite of the most determined opposition arithmetic is now taught in the day schools." In the secondary schools ciphering was taught and consisted of the fundamental operations with integers.

Sometimes women were elected to teach. A woman as much expected to teach arithmetic as Arabic or Chinese.

One of average endowment of curiosity can hardly fail to be interested, or at least amused, when he considers some of the texts in arithmetic in use in colonial days. For elementary instruction in the earliest days the Horn Book was used. It consisted of a single sheet of paper fastened to a thin piece of board and covered with a transparent piece of horn. Printed on the paper were a cross, the alphabet, a regiment of monosyllables, the Lord's Prayer, and finally, the Roman numerals. On the strength of the Roman numerals is based the claim that the Horn Book was the first mathematical primer used in America. In those days the acme of ignorance was expressed in saying, "He does not know his Horn Book," instead of the more modern expression, "He does not know his ABC's."

Of the texts treating exclusively of mathematical material, one of the first was Hodder's Arithmetic or That Necessary Art Made Most Easy, published in London, 1661. The American edition appeared in Boston in 1719. It is the first purely arithmetical text known to have been printed in America. The people of New York used a text by Peter Veniema, a Dutch school master, who died about 1612. This book was translated and printed in New York in 1730.

An English text almost as old as Hodder's and one which attained considerable circulation in America, was Crocker's, published in 1667. DeMorgan said: "It was the first arithmetic to entirely exclude demonstration, and to confine its scope to commercial questions." These characteristics made it a very popular book. It was a model for several late r writers on the subject. We read that Franklin said of himself, "Having been put to blush for my ignorance of the art of calculation, which I had twice failed to learn in school, I took Crocker's treatise and went

through it with the utmost ease." Franklin was then sixteen years old.

The book contained a picture of the author and the following poetical recommendation:

Ingenious Crocker, now to rest thou 'rt gone,
No art can show thee but thine own,
Thy rare arithmetic alone can show,
Th' vast thanks we to thy labors owe.

The first arithmetic written by an American author and printed in this country was that of Professor Isaac Greenwood of Harvard. It was probably used in his classes in college. Arithmetic was then a college subject in the senior year. So far as we know there are but two copies of this book in existence now, one in the Harvard library and one in the congressional library. The following brief description will suffice: It is a small duodecimo volume with one hundred and fifty-eight pages of text, four pages of advertisements and four pages of table of contents. The title page contans the following: Arithmetic, vulgar and decimal, with applications thereof to a variety of cases in trade and commerce, Vignette, Boston, N. E., printed by S. Kneeland and T. Isrien for T. Hancock, at the sign of the Bible and three Crowns in Ann Street, 1729. The headings of the chapters are as follows: 1, Numeration; 2, Addition; 3, Subtraction; 4, Multiplication; 5, Division; 6, Reduction; 7, Vulgar Fractions; 8,Decimal Fractions; 9, Roots and Powers; 10, Continued Proportion; 11, Disjunct Proportion; 12, Practice; 13, Rules Relating to Trade and Commerce. From the preface we take the following: "The author's design in the following treatise is to give a very concise account of such rules as are of the easiest practice in all parts of vulgar and decimal arithmetic and to illustrate each with such examples as will be sufficient to lead the learner to a full use thereof in all instances. The reader will observe that the author has inserted under all these rules where it is proper, examples with blanks for practice. This was the principal end to the undertaking; that such persons as are desirous thereof might have a comprehensive collection of all the best rules of the art of numbering with examples wrought by themselves. That nothing might be wanting to favor this design the impression is made on several of the best kinds of

paper." The paper used in this book is thick, the type large, and words and phrases which the author desired to emphasize were printed in italics. He must have regarded more than half of the book as important, for that amount is italicised.

Let us now turn to examine a book which enjoyed general popularity in the colonies. It was Dilworth's Schoolmasters Assistant, by Thomas Dilworth, first issued in London, 1741, and in America in 1769. It gave all rules and definitions in the form of questions and answers. It is a veritable catechism of confusion. In some respects it might be the grandfather of the "yes and no" method. As a sample of its contents consider the following:, taken from the twenty-second London edition:

On the single rule of three

Q. How many parts are there in the rule of three?

A. Two, single, or simple, and double, or compound.

Q. By what is the single rule of three known?

A. By three terms which are always given in the question, to find a fourth.

Q. Are any of the terms to be reduced from one denomination to another?

A. If any of the terms are of several denominations, all are to be reduced to the lowest denomination mentioned.

Q. What do you observe concerning the first and third terms?

A. They must be of the same name and kind.

Q. What do you observe concerning the fourth term?

A. It must be of the same name and kind as the second.

Q. What do you observe concerning the there terms together?

A. That the first are a supposition and the last a demand.

Q. How is the third term known?

A. By these or the like words, what cost?, how much?, etc.

Q. How many sorts of proportion are there?

A. Two, direct and inverse.

Further examination reveals other peculiarities. It consists of three parts. Part I., on whole numbers; here the pupil is led through the elementary rule of interest, partnership, exchange, double rule of three, alligation, single and double position, geometrical progressions and permutations. In Part II., after fractions the rule of three is explained again. In Part III., after decimal fractions, the rule of three is taken up a third

time. Was Mr. Walsh the real inventor of the Spiral Method in arithmetic? We are sometimes tempted to ask the same question regarding other pedagogues. To be sure we recognize the justice of re-emphasizing old methods as well as the value of new ones. Note Dilworth's treatment of interest before decimal fractions. Is it a feasible method? One writer has said that Dilworth's book is little more than a Pandora's box of disconnected rules. After all, is wholesale disapproval of the early struggles of the race altogether fair or profitable? We should, however, profit by a candid recognition of the shortcomings of our predecessors. The book appeals to the memory entirely and excludes al reasoning. In fractions, concellation is not mentioned. The book abounds in terms which we now regard as perplexing, such as; practice, conjoined proportion, alligation medial, alligation alternate, comparative arithmetic, biquadratic roots, sursolids, etc. Under duodecimals are such rules as these: Feet multiplied by feet give feet. Feet multiplied by inches give inches.

Pike's Arithmetic was another book of the same kind. In Indiana, the following incident occurred approximately a century ago. It was just after the passage of a law creating county boards of examination for teachers. This story is related by one of the candidates of that time. Pike's arithmetic was their guide council. The only question in arithmetic at this examination was: "What is the product of twenty-five cents by twenty-five cents?" The candidate says: "We had only Pikes which gave the sums and the rules. How could I tell how to do this problem when it was not in the book? The examiner thought it was $6\frac{1}{4}$ cents, but was not sure. I thought just as he did. But it looked too small to both of us. We discussed its merits for an hour or more, when he decided he was sure, and that I was qualified to teach. Accordingly, a first class certificate was issued to me."

Let us now turn our attention to the mathematical situation in the colleges of the time in question. We shall then be in a better position to understand the situation in the elementary schools. The entrance requirements to Harvard for many years after its founding in 1643, did not include even the multiplication table. Mathematics was begun in the senior year and included

arithmetic, geometry and astronomy. Let us digress to consider the contemporary situation in England. For the colonies got most of their educational ideas, at this time, from the mother country. Algebra was then unknown in America. Galileo had just passed away. Pascal and DesCartes were at the height of their careers. John Wallis was a young man. Isaac Barrow was a youth (having been born in 1630), and Newton was a babe (born 1642). An address delivered by Barrow at Cambridge some years later gives an idea of the status of mathematics at that time. From this address of Barrow, we can easily understand the reasons for the situation in our own country. Barrow said: "The once horrid names of Euclid, Archimedes, Ptolemy and Diaphantus, many of us no longer hear with trembling ears. Why should I mention the fact that by the aid of arithmetic we have learned to number the very sands. Indeed that horrible thing men call algebra, many of us, brave men that we are, have overcome, put to flight and fairly triumphed over. And very many of use have dared with straight forward glance to look into optics, and others still with intellectual rays unbroken, have dared to pierce into the still subtler and highly useful doctrine of dioptrics." If such a sentiment could be publicly expressed in England at that time, is it any wonder that common arithmetic was a senior study in the colleges of colonial America?

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ON RECENT DEVELOPMENTS IN GEOMETRY

C. D. SMITH,
A. & M. College, Mississippi

Following the introductory remarks this paper will give extracts from a previous paper read before the Louisiana-Mississippi Section of the Mathematical Association of America at a recent meeting in Natchitoches, Louisiana. In a previous article in the News Letter the writer gave three points of view in elementary geometry by way of suggesting discussion of a modified plan of presenting the subject in the secondary schools. Teachers will never agree on a detailed plan of presentation but some agreement may be reached as regards order and content. I have before me

a copy of the paper on Increased Efficiency in Geometry and Algebra, by George W. Myers, which appeared in a recent issue of School Science and Mathematics. The writer features the very important psychology of drill in presentattion of the plane geometry. I also have the report of the Committee of Ten* on the movement favoring the revision of a one year course in geometry so as to include the third dimension. This to my mind is a move of primary importance and we will surely agree that the committee headed by Dunham Jackson is representative of the best thought in present day mathematics.

In this report the battle ground is clearly defined when the question is opened as to just what shall be included in a one year course and what shall be omitted. I quote from the concluding paragraph of the report as follows. "As a means of putting into effect the proposals under discussion, the present committee suggests that a small committee, of three or five members, be appointed by the College Entrance Examination Board, to formulate appropriate requirements for tentative adoption as alternatives to the present Mathematics C and Mathematics D". These things all indicate trends that must be considered in the development of a better one year course in geometry.

The course in geometry will be determined largely by the content and structure of the text book. In any course one would expect certain important sequence theorems to be carefully demonstrated, emphasis on instruments of construction, and adequate practice exercise for the production of ability to perform in an elementary way. The problems will hinge around just what and how much may be added in the given period of time. I would like to see historic sketches inserted at proper points to indicate the fascinating origin and growth of early geometry. A large body of definitions, postulates, and theorems without proof, may be given in such a way as to give the beginner a valuable conception of the important properties of space. Algebra could be kept alive during this off year in the curriculum by putting it to work in the exercises. It may be possible to so modify the nomenclature and the statement of assumptions and definitions that the whole structure of formal geometry may be

*A Report of the Committee on College Entrance Requirements in Geometry. American Mathematical Monthly, Vol. 38, No. 5, May, 1931.

materially improved. Such a plan is suggested in the following discussion.

Beginning with Euclid many geometries have been developed presenting a variety of points of view. We are now confronted with the rise of another new geometry, or if you please another new concept of geometry. I refer to what is now known as the theory of sets of points. No attempt is made here to discuss the scope and the possibilities of this theory but rather, attention is called to some of the ideas which seem to form a basis for interpretation of various types of geometry. I am inclined to think that the basic idea suggesting the theory was to consider the ordinary line as a point set and this led to the demonstration of the continuum by Dedekind. We then have tthe Cantor continuum which does not occupy space, the Jordan curve which separates the plane, Hilbert's space filling curve, and so on. There followed definitions of various types of spaces in which certain classes of theorems and assumptions exist. The most recent summaries of the theory have been given by Chittenden*, and Lefschetz**.

We shall next consider the use of the fundamental statements and ideas of points sets in the work of elementary geometry. It would seem that such gives promise of a somewhat simplified structure for the statement of assumptions and propositions. Before discussing this point it seems well to indicate the basis of the theory of sets by the following list of definitions.

1. A point p is a limit point of a set of points E if every neighborhood of p contains a point of E distinct from p.

2. A point p is a point of condensation of a set E if every neighborhood of p contains a non-enumerable set of pints of E.

3. A set is enumerable if the points of the set may be put in a one to one correspondence with the positive integers.

4. A set is closed if it contains all of its limit points.

5. A point p is interior to E if it can be enclosed in a neighborhood containing only points of E.

6. A set is open if all points of the set are interior.

7. A set E is dense in itself if every point of E is a limit point of E.

*Symposium Lecture, Chicago, 1925.
**Colloquium Publications; American Mathematical Society, 1930.

8. A perfect set is one which is both closed and dense in itself.

9. Two sets are disjoined if they have no common point.

10. A connected set is one which cannot be separated into two disjoined closed sets.

11. Two points of E are connected if they belong to a connected sub-set of E.

12. A continuum is a perfect, connected point set.

13. A set E' of all limit points of E is called the first derived set of E.

14. A region is an open connected set.

15. A domain is a region plus its boundary.

16. A set E is compact if every infinite sub-set of E has a limit point.

17. A set E is self compact if every infinite sub-set of E has a limit point in E.

18. A set E is irreducible with respect to a property P if E, but no proper sub-set of E, has the property P.

These indicate the nature of the ideas which have given rise to the leading theorems regarding sets of points. The theory is growing rapidly and its importance is indicated by the fact that current issues of almost every mathematical journal contain contributions to the subject.

The significance of these developments rests not only with the vast extension of geometrical knowledge, but it has the very satisfying effect of suggesting a basis for unifying the concepts of the earilier geometries. Veblen* has used the properties of point sets to some extent in his Monograph on Foundatioons. The ideas are also prominent in the work of Hilbert and Forder. As an illustration of the treatment of geometry from this view point we may proceed as follows:

Let us begin with the point as an undefined concept, and assume a relation (ABC) among points to be known as order. The straight line, or linear set AB, is a point set composed of points in order (AXB),(ABX),(XAB). Distance is represented by two distinct points of a set. A ray is a set (AXB),(ABX) where point A is called the origin or end point. Two rays with the same end point are called adjacent rays and are said to form

*The Foundations of Geometry; Monographs, by J. W. A. Young.

an angle. Two linear sets with no common point are said to be parallel, or non-adjacent. A straight angle is a linear set with a point of the set as origin. A linear set with end points is called a segment. A triangle is a set composed of three angles any two of which have a common segment. Rays are said to be parallel if the corresponding sets are parallel, and the same may be said of segments. Two sets A and B are congruent if to each point of A there corresponds one and only one point of B, and vice versa. Parallel rays thru the ends of a segment form equal angles when the rays extend in opposite directions.

This list of statements may be extended to a point sufficient for the proof of any of the propositions, but these are sufficient to illustrate the idea and in particular to serve as a basis for proof of the classic theorem which we give here as an example.

Theorem: The angles of a triangle when taken together are equal to a straight angle.

Proof: Assume a triangle ABC with a linear set thru B parallel to the segment AC. One ray forms with AB an angle congruent to A and the other ray forms with BC an angle congruent to C. The two angles together with angle B determine the linear set thru B and the theorem follows.

Reference to above definitions and the construction of the figure are omitted here for obvious reasons. We merely seek to point out how geometry may be based on the idea of the point set and how the arguments follow in a very simple way. I am convinced that we can present the fundamentals of geometry more accurately and more directly by this method and in many cases we may simplify proofs. From the standpoint of the student this would be a most desirable objective and I should like to see our text books give consideration to these matters as the course is revised.

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A MATHEMATICAL RECREATION

By S. T. SANDERS, Jr.
St. Joseph, Mo.

When you held those eight spades at bridge the other night did you stop to wonder just when you could expect another such hand?

You'll have to examine approximately 200 bridge hands before eight of a kind make their appearance again.

Of these 200 you may expect 10 to be short suited, while 148 will contain three of some suit. Naturally, it is likely that none of the 200 will show 9 of a kind, since such a distribution occurs only 3 times in about 10,000 hands.

You may expect all 13 spades once every 635 billion hands. Let us know if you fail to get them.

As a starting point for these calculations let us inquire into the total number of hands possible. That is, the number of ways of choosing 13 things from 52 things. This is by definition, $C(52,13)=635{,}013{,}559{,}600$.

Now of these hands, how many will contain exactly three spades? For each distinct set of three spades, there are possible $C(39,10)$ distinct combinations of hearts, clubs, and diamonds, there being 39 remaining cards to be taken 10 at a time. But there are $C(13,3)$ total spade combinations, hence $C(13,3).C(39,10)$ hands containing three spades.

Or, in general, we have $K_n=C(13,n).\ C(39,13\text{-}n)$ (1) where K_n is the number of hands containing n spades. Naturally indentical formulas hold for the other suits.

From this formula results the following table:

n	K_n (number of hands with n spades)	No. of such hands per 1000
0	8,122,425,444	13
1	50,840,366,668	80
2	130,732,371,432	206
3	181,823,183,256	286
4	151,519,319,380	239
5	79,181,063,676	125
6	26,393,687,892	41
7	5,598,661,068	9
8	740,999,259	1
9	58,809,465	0
10	2,613,754	0
11	57,798	0
12	507	0
13	1	0
Total	635,013,559,600=C(52,13)	1000

However, of greater interest than the frequency of n of a particular suit is the frequency of n of any suit.

Let us see how many of the 635 billion hands have 13 of a kind. Obviously 4, since there is one such spade hand.

The following formula evidently holds for the indicated values of n:

$$N_n = 4K_n \qquad (7 \leq n \leq 13) \qquad (2)$$

where N_n is the number of hands containing n of any suit.

For $n=5$, 6 this formula does not hold, since there will be hands containing 5, 6 of each of two suits. Such hands would be counted twice by (2).

Formula (3) eliminates the duplication:

$$N_n = 4K_n - C(26, 13-2n).C(13, n).C(4,2).(n=5, 6) \qquad (3)$$

Assuming n spades and n hearts, $C(26, 13\text{-}2n)$ gives the number of club and diamond assortments possible to complete the hand. Now there are $C(13, n)$ combinations of the n spades, and likewise $C(13, n)$ of the n hearts, and finally, for each of the $C(26,13\text{-}2n)$. $C(13,n)^2$ hands of spades—hearts, and clubs—diamonds, there exist $C(4,2)$ arrangements of the suits. That is, n spades, n clubs might occur, etc.

However, for $n \leq 4$ formula (3) also fails, since there occur hands with n of each of three suits, thus again yielding duplication. For these cases we resort to:

$$N_n = K_n - C(26,13\text{-}2n).C(13,n)^2.C(4,2) + C(13,13\text{-}3n).C(13,n)^3.C(4,1) \qquad (0 \leq n \leq 4) \qquad (4)$$

The last term will be seen to represent the number of hands with n of each of three suits. Such hands, however, are counted three $C(3,2)$ times in the second term, which gives the number of hands with n of each of two suits. That is, the second term counts a hand containing one spade, one heart, one club, and ten diamonds as a spade-heart hand, again as a spade-club hand, and finally as a heart-club hand.

Hence the number of hands containing n of exactly two suits is

$$C(26,13\text{-}2n).C(13,n)^2.C(4,2) - 3C(13,13\text{-}3n).C(13,n)^3.C(4,1).$$

Now $4K_n$ counts these latter hands twice—once for each of

the two suits. And $4K_n$ counts the n—3—suited hands three times. Hence,

$N_n = 4K_n$ - [$C(26,13\text{-}2n)$. $C(13,n)^2$. $C(4,2)$ - 3 $C(13,13\text{-}3n)$. $C(13,n)^3$. $C(4,1)$] - 2 $C(13,13\text{-}3n)$. $C(13,n)^3$. $C(4,1) = 4K_n$— $C(26,13\text{-}2n)$. $C(13,n)^2$. $C(4,2)$ + $C(13,13\text{-}3n)$. $C(13,n)^3$.$C(4,1)$

$(o \leq n \leq 4)$

Formulas (2), (3), (4), yield the following results:

n	N_n (number of hands with n of any suit)	No. of such hands per 1000
0	32,427,298,180	51
1	195,529,653,800	306
2	412,128,237,456	649
3	471,366,136,384	742
4	423,314,340,020	667
5	290,884,898,304	458
6	105,115,385,232	166
7	22,394,644,272	35
8	2,963,997,036	5
9	235,237,860	0
10	10,455,016	0
11	231,192	0
12	2,028	0
13	4	0

THE TRIGONOMETRY BASED ON A CENTRAL CONIC

By H. L. SMITH,
Louisiana State University

In this NEWS LETTER* has appeared a series of interesting papers by B. E. Mitchell on what he calls sextantal trigonometry. The treatment in these papers is similar to the usual treatment of the ordinary trigonometric functions. A simple method of treatment of the circular and hyperbolic functions, based on the notion of area of sector, has been given by the writer**. It is the object of this note to show that the latter

*This NEWS LETTER, Vol. 5, Nos. 1, 2, 5.
**This NEWS LETTER, Vol. 5, No. 4.

method when applied to a general central conic yields the definitions and properties of a class of functions which include as special cases the circular functions, the hyperbolic functions, and the sextantal functions of Mitchell.

§1. **Basic formulas.** In the paper of the writer cited above it was shown how to define, by means of a certain type of curve C, two single-valued functions $x(t), y(t)$. The curve C was such that (1) it was continuous and connected; (2) it was cut in at most one point by every half-line with the origin as end-point; and (3) it was cut by the positive x-axis in one point, $(a,0)$ say.

Now suppose the equation of C is

(1) $f(x,y)=0,$

where f has continous first partial derivatives f_x f_y .
Then assuming that $dx|dt$, $dy|dt$ exist, we have

(2) $f_x\ (dx/dt)+f_y\ (dy/dt)=0.$

But by § 2 of the paper cited above,

(3) $y(dx/dt)—x(dy/dt)=—1.$

On solving (2), (3) simultaneously for dx/dt, dy/dt we get

(4) $dx/dt=—f_y/(xf_x+yf_y),\ dy/dt=f_x/(xf_x+yf_y).$

The above derivation assumes the existence of the derivatives sought; but by the increment methods, as in the paper cited, the formulas (4) may be derived without this assumption.

§2. **The homogeneous case**. If

(5) $f\ (xy)=\mathrm{F}(x,y)—1=0.$

where F (x,y) is a homogeneous polynomial of degree n, then

$$xf_x+yf_y=n\mathrm{F}$$

and (4) becomes

(6) $dx/dt=—(1/n)f_y,\ dy/dt=(1/n)f_x.$

§3. **The central conic case.** Let us apply the above to the case in which

(7) $f\ (xy)=x^2+2kxy+ly^2—1=0)$ (l-k^2 not$=0$)

which represents a central conic passing through the point (1,0). The conic is an ellipse if l-$k^2>0$, an hyperbola if l-$k^2<0$; in the latter case we restrict the locus to that branch of the hyperbola which crosses the x-axis at the point (1,0).

The functions $x(t), y(t)$ we call the conic cosine and conic sine, respectively and denote them by cosc t, sinc t, or more explicitly by, cosc $(t,k,l,)$, sinc (t,k,l), respectively. We proceed to find some of their properties.

We have at once

(8) $\operatorname{cosc} 0=1,\ \operatorname{sinc} 0=0.$

Moreover by (6)

$$(10)\quad \begin{cases} (d/dt)\operatorname{cosc} t=-(k\operatorname{cosc} t+l\operatorname{sinc} t)\\ (d/dt)\operatorname{sinc} t=\operatorname{cosc} t+k\operatorname{sinc} t\end{cases}$$

In the elliptic case, the functions are periodic:

$$(11)\quad \begin{matrix}\operatorname{cosc}(t+2np)=\operatorname{cosc} t\\ \operatorname{sinc}(t+2np)=\operatorname{sinc} t\end{matrix}\qquad (l^2-k^2>0)$$

where $p=\pi/\sqrt{l-k^2}$.

We now introduce four other conic functions, corresponding to the ordinary tangent, cotangent, secant, and cosecant, defined by the following equations:

$$(13)\quad \begin{cases}\operatorname{tanc} t=(\operatorname{sinc} t)/(\operatorname{cosc} t),\ \operatorname{ctnc} t=(\operatorname{cosc} t)/(\operatorname{sinc} t)\\ \operatorname{secc} t=1/\operatorname{cosc} t,\ \operatorname{cscc} t=1/\operatorname{sinc} t\end{cases}$$

From (10), (13) we have at once

$$(14)\quad \begin{cases}1+2k\operatorname{tanc} t+l\operatorname{tanc}^2 t=\operatorname{secc}^2 t\\ \operatorname{ctnc}^2 t+2k\operatorname{ctnc} t+l=\operatorname{cscc}^2 t\end{cases}$$

Also by (9), (13)

$$(15)\quad \begin{cases}(d/dt)\operatorname{tanc} t=\operatorname{secc}^2 t\\ (d/dt)\operatorname{ctnc} t=-\operatorname{cscc}^2 t\\ (d/dt)\operatorname{secc} t=l\operatorname{tanc} t\operatorname{secc} t+k\operatorname{secc} t\\ (d/dt)\operatorname{cscc} t=-\operatorname{ctnc} t-k\operatorname{cscc} t\end{cases}$$

We consider now the question of addition theorems.

§4. Addition theorems for the conic functions. The truth of the equation

(16) $\operatorname{cosc}(a-b)=\operatorname{cosc} a\operatorname{cosc} b+2k\operatorname{cosc} a\operatorname{sine} b+l\operatorname{sinc} a\operatorname{sinc} b$

is indicated by the fact that it reduces to (10) when $a=b=t$. To prove it, set

$g(t)=\operatorname{cosc}(a-b+t)\operatorname{cosc} t+2k\operatorname{cosc}(a-b+t)\operatorname{sinc} t+l\operatorname{sinc}(a-b+t)\operatorname{sine} t,$

and note that (16) is equivalent to

(17) $g(1)=g(0)$

But (17) is true; for by aid of (9) we find $g^1(t)=0$ so that $g(t)$ is a constant function of t.

If (16) is differentiated with respect to b by aid of (9) and (16) is applied to the result, there results

(18) $\operatorname{sinc}(a-b)=\operatorname{sinc} a\operatorname{cosc} b-\operatorname{cosc} a\operatorname{sinc} b$

On putting $a=0$ in (16), (18), we obtain

$$(19)\qquad \begin{cases} \operatorname{cosc}(-b)=\operatorname{cosc} b+k \operatorname{sinc} b \\ \operatorname{sinc}(-b)=-\operatorname{sinc} b \end{cases}$$

Moreover by replacing b by $-b$ in (16), (18) and using (19), we get

$$(20)\qquad \begin{cases} \operatorname{cosc}(a+b)=\operatorname{cosc} a \operatorname{cosc} b-l \operatorname{sinc} a \operatorname{sinc} b \\ \operatorname{sinc}(a+b)=\operatorname{sinc} a \operatorname{cosc} b+\operatorname{cosc} a \operatorname{sinc} b \end{cases}$$

Finally

$$(21)\qquad \operatorname{tanc}(a+b)=\frac{\operatorname{tanc} a+\operatorname{tanc} b}{1-l \operatorname{tanc} a \operatorname{tanc} b}$$

§5. **Conclusion.** It is plain that if we take $(k,l)=(0,1)$ we get the circular functions; if we take $(k,l)=(0,-1)$ we get the hyperbolic functions; and if we take $(k,l)=(\frac{1}{2},1)$ we get the sextantal functions of Mitchell, except that we have a different unit of measure for t. His unit is analogous to the right angle, while our unit is analogous to the radian.

ON GENERALIZATION AND CORRELATION THROUGH MATHEMATICS

By W. PAUL WEBBER,
Louisiana State University

The history of Differential Equations furnishes a striking example of the advantage of mathematical symbolism. It was Leibnitz who first used the integral sign as we know it; and with it came all the advantage of a convenient symbolism in a new subject. The acrimonious controversy into which Newton and Leibnitz were led by their friends created an atmosphere which deprived British mathematicians of the use of this new symbolism for a long time. This was a great handicap to British mathematicians and gave Continental mathematicians a distinct advantage. Often it happens for different reasons that students of science deprive themselves of the use of the most powerful of methods by avoiding mathematics as a preparation for the study of science. They do not at first specially care for the strict thinking required in mathematics. They soon lose interest and discontinue the study at the earliest permissible date. They do not realize that they are making their future studies more dif-

ficult and giving themselves a serious handicap in the race for success.

About thirty years ago the author of a college text on General Physics made the statement in his preface to the effect that he attempted as far as possible to use the thought processes of calculus without its symbolism. He said he was thus preparing the student for calculus in a certain sense. This author recognized the value of the mathematical method but could not use it in its full power because his pupils had not the necessary mathematical basis at the time the course in physics must be given. The author of another text in General Physics published within a few years of the time the one previously mentioned came off the press, said in his preface that "Calculus is the natural language of physics." He did not however venture to use much calculus in his book. The situation today, for some reason, is not much different. I suspect the reason lies largely in the conservative attitude of the teachers of mathematics and physics. I, in my own mind, cannot justify the condition.

In 1894 Professor John Perry wrote—"Much has been written about the correlation of the physical sciences; but when we observe a young man who has worked almost altogether at heat problems, suddenly show himself acquainted with most difficult investigations in other departments of physics, we may say the correlation of the physical sciences lies in the equation of continuity

$$\frac{dw}{dt}=a^2\left(\frac{d^2w}{dx^2}+\frac{d^2w}{dy^2}+\frac{d^2w}{dz^2}\right)$$

Indeed, Professor Webster wrote a few years ago in one of his books that there are only eight fundamental equations in mathematical physics and the all these fall into only two really distinct types.

There are equally striking examples of such correlation in more elementary fields of science. I will cite some examples of one case.

(a) The rate of increase in the number of bacteria in the presence of an abundant supply of culture is proportional to the number present at the time under consideration. This law of nature (for it is a law of nature) is concisely and exactly ex-

pressed by the differential equation,

(1) $$D_t z = kz.$$

where z is the number of bacteria, t the time and k a constant for a given case.

(b) The rate of change in a chemical reaction is proportional to the molecular density (or concentration) of the reacting substance. This law is expressed precisely by the differential equation,

(2) $$D_t z = kz.$$

where z is the molecular density, t the time and k a constant for a given substance. It will be noted that to the mathematician this equation is identical with the previous one. But in the last case the variables have a different concrete meaning from those in the first equation.

(c) Consider a tank of salt solution of molecular concentration z. The solution is to be diluted by passing fresh water through the tank at a constant rate v, the water being thoroughly mixed with the salt solution as the liquid flows through. The equation that expresses the process is

$$D_t z = -kz.$$

It will be noticed that here we have the same equation as before except for a sign. To use Professor Perry's comparison, it is easy to see how a man who has worked exclusively with problems on diluting salt solutions might suddenly become familiar with certain problems in biology or chemistry. He would only have to learn the meaning of the new terms and give these names to the variables in the same old equation. Here is illustrated how mathematical method can correlate, at quite an elementary stage of study, what seem to be widely divergent sciences.

It is not difficult to show freshmen that a function whose rate of change is proportional to itself is the exponential function and that such a function satisfies the above differential equation.

The question is raised as to whether freshmen can be taught sufficient calculus to enable them to absorb a course in physics

in which a modest amount of calculus is used. My answer is in the affirmative. I believe it is better to omit some of the classic material from algebra and a little from trigonometry in the first year to make room for more than the present useless modicum of calculus found in texts in college algebra.

In my humble opinion the proper place to teach determinants is at the beginning of the standard first course in elementary analytic geometry. For it is here that they have one of their most useful and natural applications. Other topics may be pared a little until there is enough space and time to give a really useful introduction to the methods of calculus, both differential and integral. This first glimpse will be an eye opener and a stimulant to many and will be as intelligible as any mathematics at all to others. Such a plan would early remove the idea of many that higher mathematics is just more and harder algebra and geometry of the same general kind that they had in high school. I find it no harder to get pupils to understand the first elements of calculus than it is to get them to understand some of the things that are in the traditional courses. The new power gained in the ability to grasp and correlate the simpler laws of nature would amply justify the plan. For those who are to take a regular course in calculus, this early "shove off" will enable them to make more rapid progress from the start and the time gained can be used to make up any important matter that was omitted from the first year. A new application may at this time be given to such material.

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THE NOTION OF AREA OF SURFACE

By H. L. SMITH,
Louisiana State University

The notion of area of surface is one which still presents certain difficulties. For this reason a brief summary of the present state of the subject may be useful.

In the present paper the most promising definitions given so far will be stated in geometric form. This will necessitate considerable modification in the case of some of them, in-as-much as the original definitions were given analytically. They will

also be given in such form as to avoid preliminary assumptions about the surface. By surface we shall mean a two-cell (to be defined below) and shall not consider definitions which apply to special kinds of two-cells only. This will exclude certain important and interesting work, in particular, that of Tonelli, which applies only to surfaces whose equations can be written in the form $z=f(xy)$. However, these papers will be listed at the end of the article along with the others consulted.

§1. The revised classical definition. Throughout this paper let s denote a closed 2-cell, that is, a set of points which can be put into one-to-one continuous correspondence with a closed square. Let p be a polyhedron inscribed in s, that is, having each of its vertices on s. Suppose also that the boundary of p is a polygon inscribed in the boundary of s and that each face of p is a triangle. We denote also the area of p by the letter p. By the *norm* of p we shall mean the length of its longest edge.

Now it would be natural to suppose that the area of s (if existent) would be defined as the limit as the norm of p decreases of the area of p. Such indeed was the classical definition and standard treatises once contained proofs that this definition leads to the usual formula for area of surface. However H. A. Schwartz showed that these proofs are incorrect; for he showed that in as simple a surface as a cylinder can be inscribed a polyhedron p of arbitrarily small norm with arbitrarily great area. This he did by constructing p so that each of its faces was almost perpendicular to the surface of the cylinder.

The work of Schwartz was later duplicated by Cartan.

The examples found by Schwartz and Cartan suggest the following modified form of the classical definition. Let $C_r(s)$ denote the greatest lower bound of the areas of all p of norm$\leq r$. Denote by $C(s)$ the least upper bound of $C_r(s)$. In case $C(s)$ is finite, we define it to be the area of s in the revised classical sense. It is clear, since $C_r(s)$ is a monotonic function of r, that $C(s)$ is also the limit of $C_r(s)$ as r approaches zero.

It has been proved by the writer in a paper not yet published that the following is true.

Theorem. If the equations

s: $x=x(u,v)$, $y=y(u,v)$, $z=z(u,v)$.

define a one-to-one, continuous correspondence between a 2-cell s and the square Q: $0 \leq u \leq 1$, $0 \leq v \leq 1$,

and if the partial derivatives $x_u, y_u, z_u, x_v, y_v, z_v$,

are continuous and such that

(1) $A^2+B^2+C^2$ not$=0$ on Q,

where

$$A=y_u z_v - z_u y_v,\ B=z_u x_v - x_u z_v,\ C=x_u y_v - y_u x_v,$$

then C(s) exists and

(2) $C(s)=\iint_Q \sqrt{A^2+B^2+C^2}\, du\, dv.$

The proof is complicated and no indication of its nature can be given here.

Rademacher has obtained a result which is of interest in this connection. He has shown that if $x(u,v)$, $y(u,v)$, $z(u,v)$ satisfy Lipschitz conditions, then s has a tangent plane at every point, except for a set of points whose projection on each of the coordinate planes is a set of measure zero. The integral of (2) will thus exist in the sense of Lebesgue. Moreover that integral is the limit of a properly chosen sequence of polyhedra p with norms having limit zero. It would follow in this very general case that $C(s) \leq \iint_Q \sqrt{A^2+B^2+C^2}\, du\, dv$. It is an open question whether the equality sign holds in all cases.

§2. **The modified definition of Schwartz.** The work of Schwartz and of Rademacher suggests another possible definition of area of surface, which may be stated as follows. Corresponding to the 2-cell s let p_n be a sequence of polyhedra of the type of §1. Let the limit of the norm of p_n be zero as n approaches infinity. Also let the greatest lower bound of a_n be positive, where a_n is the smallest sine of the face angles of p_n. If then the limit of p_n exists for every such sequence p_n and has the same value for all of them, we denote its value by S(s) and call it *the area of s in the sense of Schwartz.*

§3. **The modified Lebesgue definition**. In his dissertation Lebesgue has given a definition which is probably equivalent to the one about to be given. We need first some preliminary notions.

A being any set, we denote by V_r A the set of all points

distant by not more than r from A. Then, A and B being any two sets we define E(A,B), the *ecart* of A,B, as the greatest lower bound of all values of r such that A includes B and B includes A.

Now if s is any 2-cell and p_n is a sequence of triangular polyhedra (not necessarily inscribed in S) such that (1) the limit of the norm of p_n is zero, (2) the limit of $E(p_n,s)$ is zero, (3) the limit of p_n exists, then we define $L(s)$ to be the greatest lower bound of the limit of p_n for all such sequence p_n, and call $L(s)$ *the area of s in the sense of Lebesgue.*

§4. The modified de la Vallee-Poussin definition. De la Vallee-Poussin has given a definition of area of surface based on the idea of breaking the surface up into pieces, projecting each of the pieces on to the plane for which that projection has a maximum area, adding the areas of their maximum projections and passing to the limit as the greatest of their areas approaches zero. But considerable care is necessary in formulating this definition precisely. For even in the simple case of a plane square, if care is not taken in breaking up the square into pieces, the pieces may not have area even in the sense of Lebesgue. Even if the pieces are taken to be closed and allowed to overlap along boundries, in which case each piece has Lebesgue measure, the sum of the Lebesgue measure of the pieces may not equal the area of the square.

The following is an attempt at a satisfactory formulation of de la Vallee Poussin's definition. If s is any 2-cell we define $P(s)$ as the least upper bound of the Lebesgue measures of the projections of s on all possible planes. We also define $p(s)$ as the least upper bound of the Lebesgue measure of the interiors of the projections of s on all possible planes. Now set

(3) $s=s_1+\ .\ .\ .\ .\ .\ .\ +s_m$

where s , ... , s_m are closed 2-cells non-overlapping except for boundary points. Let $V_r\ (s), v_r\ (s)$ be respectively the greatest lower bound of $P(s_1)+\ .\ .\ .\ +P(s_m)$ and the least upper bound of $p(s_1)+\ .\ .\ .\ +p(s_m)$ for all partitions (3) of s of norms $\leqq r$, the norm of (1) being the maximum diameter of the cells $s_1\ .\ .\ .\ s_m$. Then let $V\ (s), v(s)$ be the least upper bounds of $V_r\ (s)$, (s), v_r, respectively. Then if $V(s)=v(s)$, we call $V(s)$ **the area of s in the sense of de la Vallee Poussin.**

§5. **The modified Jansen definition.** In his Konigsberger dissertation, O. Jansen gave a definition of area of surface which differs from that of de la Vallee-Poussin in two respects. Instead of dividing s into 2-cells, Jansen effects a partition of s into closed sets by means of a division of the whole space into cubes; also instead of taking the maximum projection of each part of s, he projects each part on to three mutually perpendicular planes and takes the square root of the sum of the squares of the Lebesgue measures of the projections. We leave the precise formulation to the reader.

§6. **The modified Young definition.** W. H. Young has given a definition of area of surface based on what he calls the nation of "area of a skew curve", which we prooceed to define.

Let $P_1, \ldots, P_n$ be n points in a plane and let $(x_1\ y_1), \ldots, (x_n\ y_n)$ be their rectangular coordinates in any cartesian system in that plane. Then the expression.

$$|\tfrac{1}{2}(y_1+y_2)\ (x_2-x_1)+\tfrac{1}{2}(y_2+y_3)\ (x_3-x_2)+\ \ldots\ldots+\tfrac{1}{2}(y_n+y_1)\ (x_1-x_n)|$$

is independent of that coordinate system and will be denoted by $A(P_1, \ldots, P_n)$. Here obviously $A(P_1 \ldots P_n)=A(P_n, \ldots, P_1)$. Now let $P_1 \ldots P_n$ be n points not necessarily in the same plane. Let $U_1 \ldots U_n, V_1 \ldots V_n, W_1 \ldots W_n$ be their respective projections on three mutually perpendicular planes, and let $A(P_1 \ldots P_n)$ be the square root of the sum of the squares of $A(U_1 \ldots U_n)$, $A(V_1 \ldots V_n)$, $A(W_1 \ldots W_n)$. Then $A(P_1 \ldots P_n)$ is independent of the choice of the three mutually perpendicular planes, and the present definition does not contradict the previous one. Finally let C be a simple closed space curve. Let $P_1 \ldots P_n$ be consecutive points on C and let d be length of the longest of the chords $\overline{P_1 P_2}, \ldots\ldots, \overline{P_{n-1} P_n}, \overline{P_n P_1}$. Then let $A(C)$, $a(C)$ be respectively the upper and lower limits as d approaches zero of $A(P_1 \ldots P_n)$. We call them in case they are equal merely the **area of** C.

We are now in a position to give Young's definition, again making certain changes in order to avoid difficulties similar to those which beset the original definition of de la Vallee-Poussin. To this end, set

$$s=s_1+\ \ldots\ +s_n$$

where $s_1 \ldots s_n$ are 2-cells, non-overlapping except for boundary points. Let $c_1, \ldots . c_n$ be the boundaries of $s_1, \ldots, s_n$ respectively. Let $Y_r(s), y_r(s)$ be respectively the least upper bound of $A(c_1) + \ldots + A(c_n)$ and the greatest lower bound of $a(c) + \ldots + a(c_n)$. Let $Y(s), y(s)$ be the least upper bounds of $Y_r(s), y_r(s)$, respectively. If $Y(s) = y(s)$, we call $Y(s)$ **the area of s in the sense of Young.**

Young has given conditions under which his definitions lead to the usual integral for the area, and Burkhill has given conditions under which the definition leads to a more general integral. Burkhill claims that his results are more general than Lebesgue's.

§7. **The Caratheadory definition.** Caratheadory has given a definition of area of surface which applies not only to surfaces but to certain other sets as well. (The same was true of Jansen's definition.) We give it for a 2-cell.

If c is a convex set, let $D(c)$ be the least upper bound of the Lebesgue measures of the projections of c on all possible planes. Now let s be a 2-cell and let $c_1, \ldots, c_n$ be convex sets such that s is included in $c_1 + \ldots + c_n$. Let d be the largest of $D(c_1), \ldots, D(c_n)$. Then let $K_r(s)$ be the greatest lower bound of $D(c_1) + \ldots + D(c_n)$ for all such $c_1 \ldots . c_n$ for which $d \leqq r$. Finally let $K(s)$ be the least upper bound of $K_r(s)$. We define $K(s)$, if finite, as **the area of s in the sense of Caratheadory.**

§8. **The Minkowski definition.** One of the first definitions to be given was that of Minkowski, which like those of Jansen and Caratheadory applies to other sets in addition to surfaces. The definition is easily stated. If the limit of the Lebesque measure of $V_r s$ divided by $2r$ exists as r approaches zero, we denote it by $M(s)$ and call it **the area of s in the sense of Minkowski.**

Sibirani has shown that under rather strong hypotheses the definition of Minkowski leads to the usual integral.

§9. **Conclusion.** We have now finished our survey of the definitions of area of surfaces given in the literature. We have revised all of them except those of Caratheadory and Minkowski. We now have them in such form that they apply to an arbitrary

2-cell. Numerous questions at once arise. To what extent are these definitions equivalent? Which of the familiar theorems about length of arc admits of extension to area of surface? In particular does the existence of area of surface imply anything about the existence of tangent planes to the surface? The writer hopes to obtain the answers to some of these questions.

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PROBLEM DEPARTMENT

Edited by T. A. BICKERSTAFF,
University of Mississippi

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Problems and solutions will be credited to their authors.

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No. 3—Proposed by L. E. Scally, Morse, La.:

To construct an athletic track ¼ mile long in the form of an ellipse with major axis equal twice the minor axis.

No. 4—Proposed by the Editor:

Express the square root of $18+25\sqrt{14}+18+2\sqrt{63}$ as the sum of three square roots.

No. 5—Proposed by R. M. Guess, University, Miss.:

If a cow is tied with a 75-ft. rope to the corner of a barn 25 ft. square, over how much tterrittory can she graze?

No. 6—Proposed by Geo. A. Garrett, University, Miss.:

Find the length of the Hyperbola $X^2—Y^2=1$ from a vertex to a point (x_1y).

SOLUTIONS

No. 1—Proposed by the Editor. Solved by Richard H. Stewart and the Editor.

Find a finite number, x, so that the x-th power of x is a minimum:

Let $y=x^x$

$\log y=x \log x$

$$\frac{y'}{y}=\frac{x\ x'}{x}+\log x$$

$$=1+\log x$$

To get either maximum or minimum put $y^1=o$.

Then $1+\log x=o$

and $x=\dfrac{1}{e}$ $\qquad$ y'' *is* $>o$ for $x=\dfrac{1}{e}$.

Hence $\dfrac{1}{e}$ is a minimum for x.

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